is that particle drift motion must conserve energy so that a drift trajectory must be a contour of constant total energy, and calculation of these contours is much faster and more efficient than integration of forces. If there are no inductive electric fields due to a changing magnetic field, $\partial B/\partial t = 0$, then we can express the electric field as the gradient of a scalar, U. Assuming that the drift speed is insignificant compared to gyration, we can write the total energy as, $W = KE + PE = mv^2/2 + qU = \mu B_m + qU$, where $B_m(K)$ is the magnetic field magnitude at the mirror point. We label particles by their μ and K values, where $K = J/\sqrt{2m\mu} = 4\int_0^m \sqrt{B_m - B(s)}ds$. So then, rather than considering drift motion only in the equatorial plane, we calculate the iso-energy contours on a mirror point K-surface, and then map to the equatorial plane.

Whipple showed that one can greatly simplify the classification of drift orbits by a coordinate transformation which relies on the conservation of energy; i.e., since $dW/dt = 0 = d(\mu B_m)/dt + d(qU)/dt$, then

$$\frac{\partial U}{\partial B_m(K)} = \frac{-\mu}{q} \tag{1}$$

Which says that in UB(K) space, particle trajectories are straight lines whose slope is proportional to $-\mu/q$. For a simple dipole magnetic field with a Volland-Stern electric field (Figure 1), the mapping from real space to UB(K) space is double valued. However, this can be resolved by splitting the magnetosphere into night and day halves, the boundary being the line at which contours of constant B and U are tangent. In the night half of the magnetosphere the particles convect toward higher B (earthward), while on the day side particles convect toward lower B (anti-earthward), easily distinguishing the two populations. When a particle orbit crosses a tangency line in real space, it reverses direction in UB(K) space so that these tangency lines are also limits of motion in UB(K) space. The ionosphere of the earth and the magnetopause complete the bounding of particle motion in UB(K) space.

With this mapping, topological boundaries become simple geometrical constructions involving slopes and tangents, which can be easily automated and calculated to any level of precision. For example, the plasmapause is the last closed drift orbit for a zero energy ion. This translates to a horizontal line in UB(K) space that is tangent to the peak of the lower bounding tangency curve. Likewise the Alfvén layer for any energy is found by picking an energy (slope) that intersects the tail and making it tangent to the appropriate bounding curve such that it maximally penetrates into the magnetosphere.

One must recalculate this mapping for each value of K, or equatorial pitch angle desired. In general the tangency curves can be very different, even changing the topology of the drift boundaries in UB(K) space. These bounce averaged drift orbits are then mapped in real space along the field lines to the equatorial plane to compare trajectories of differing K (Figure 2).

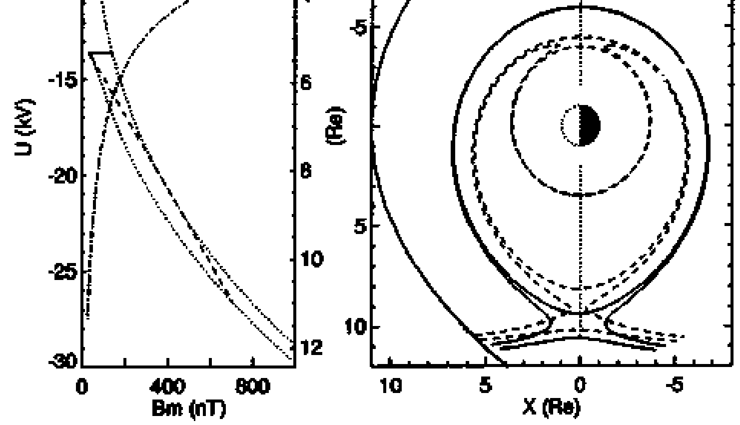

Fig. 1: An equatorial cut in the X-Y plane to UB(K) mapping for a dipole magnetic field + Volland-Stern [*Volland,* 1973, *Stern,* 1973] shielded electric field appropriate for K_p=0 [*Maynard and Chen,* 1975]. Lines of tangency between contours of constant B and U are shown dotted. Dash-dotted line converts B_m to radial distance with right hand scale. The plasmapause, the last closed 0.0 eV/nT trajectory, is bracketed by solid lines. The open drift trajectory that penetrates most deeply into the magnetosphere is bracketed by dashed lines. With a slope of 19.2 eV/nT, it follows a banana orbit but only crosses a tangency line once, at dusk close to the earth.

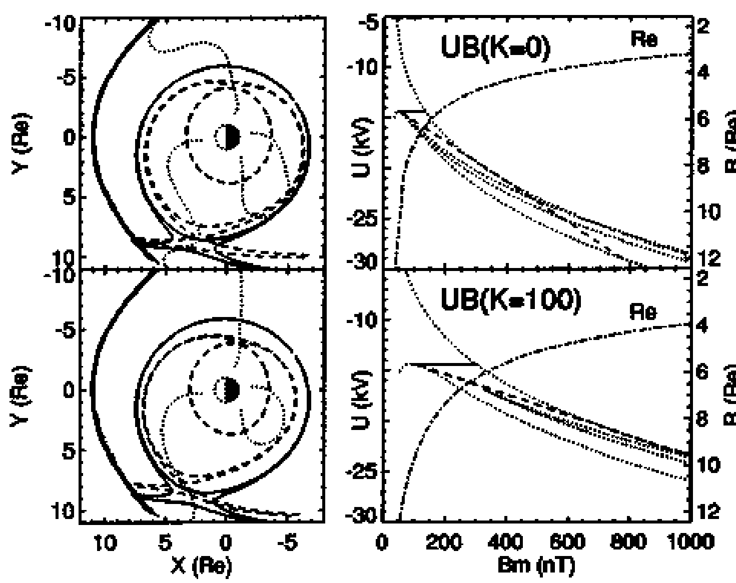

Fig. 2: Equatorial X-Y to UB(K) mapping for realistic Olson-Pfitzer [*Olson et al.,* 1979] magnetic field and Volland-Stern (K_p=0) + ionospheric dynamo [*Richmond et al.,* 1980] electric fields. Top panels calculated for 90° pitchangles, K=0; bottom panels for K=100. The extra loop in X-Y tangency curves generated by a quadrupolar ionospheric field becomes a topological pleat in UB(K) space. Trajectories labelled similarly as previous figure. Dashed trajectory has a slope of 10.2 eV/nT in top panels, 20.2 eV/nT in bottom panels.