The Quadrupole as a Source of Cusp Energetic Particles: I. General Considerations

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The recent discovery of energetic particles trapped in the cusp (CEP) in close association with diamagnetic cavities (CDC) strongly suggest a new location for particle acceleration. Such an acceleration region has long been sought as the mysterious source of outer radiation belt electrons (ORBE) and ring current ions (RCI). In this paper we present some theoretical support for the acceleration mechanism and estimate that the quadrupole cusp may be the most powerful accelerator in geospace. If so, then the CEP/CDC may provide a useful predictor of the injection of ORBE/RCI fluxes that are detrimental to geosynchronous spacecraft.

1. INTRODUCTION

The 1959 discovery that the Earth had radiation belts came as a great surprise to the Van Allen's scientific team, as Ludwig quipped, “My God, space is radioactive!” (*Hess68*). It was a surprise because none of the known inputs--cosmic rays, the Earth's ionosphere, the Sun's atmosphere--possessed such an energetic particle flux, making their origin mysterious. The first radiation belt source discovered posited that cosmic rays striking the upper atmosphere would sputter neutrons up into space, where spontaneous neutron decay would produce charged protons and electrons that could become trapped in the magnetic field. By this the origin problem was conveniently moved to the problem of cosmic ray production, itself a somewhat open issue. But this source only explained the origin of the inner radiation belts around 2 Re, not the origin of the Outer Radiation Belt Electrons /Ring Current Ions (ORBE/RCI) at 5 Re.

Forty years later, it is remarkable that we still have not identified the origin of ORBE/RCI (e.g., McIlwain's mechanisms 1 & 5 (*McIlwain96*), despite numerous spacecraft that have explored nearly all the near Earth space. Several factors contribute to this difficulty, including the variability of the ORBE population, and the properties of a dipole trap. Accordingly, there have been several notable attempts to explain ORBE as of Jupiter origin (*Baker86)*, substorm produced (*Ingraham99)*, recycled through the magnetosphere (*Nishida76*), wave-particle resonant acceleration (*Elkington99*), shock-accelerated (*Li93, Reeves98)*, CME related (*Baker98*) or due to magnetic clouds (*Farrugia98*). All of these explanations fail to correlate well with satellite measurements of ORBE (*Paulikas79*).

Similar difficulties attend the explanations for RCI, which posit warm ($E > 10$ keV) ions in the plasmasheet adiabatically diffusing into the dipole trap region. Since the tail is vast, magnetic mapping uncertain, and reconnection certain, the origin of this precursor warm power law tail on the plasmasheet populations (*Christon89*), is often brushed under the rug, however, the ubiquity of tracer species throughout the magnetosphere, and their rapid rise during active periods make “tail theories” of their origin suspect, since they would not have sufficient time to traverse the long distances of the tail (*Sheldon00*).

Thus it came as a big surprise when POLAR found trapped MeV ions and electrons in the distant cusp over the Earth's poles with nearly exactly the same phase space density as the ORBE/RCI, in what were then named cusp energetic particle (CEP) events (*Sheldon98, Chen97, Chen98a, Chen98b, Fritz99*). Was this population part of a migrating Nishida circulation? Does it map to the tail (*Delcourt99*)? Did it come from the ORBE/RCI population (leakage), or is it supplying the ORBE/RCI population (*Chang98, Trattner01*)? Further analyses (*Sheldon00, Chen00, Chen01a, Chen01b, Chen02, Fritz00, Fritz03a, Fritz03b, Sheldon03*) suggested that we had serendipitously stumbled into the nursery where ORBE/RCI are born, where low-energy ($E<10$ keV) plasma is accelerated to energetic particle status ($E>100$ keV), where non-Maxwellian power-law “tails” were added to the thermalized particle distributions. Furthermore, if the mechanism is a simple consequence of topology, of a quadrupole trap, then there is no reason not to believe that such nurseries exist in all magnetic cusp geometries, including that of the heliosphere and the galaxy. If so, then this mechanism may also explain the ubiquitous source of cosmic rays (*Fermi49, Alfvén49, Alfvén50*), bringing theoretical closure to the origin of the inner radiation belts as well. Although this book is an ideal place to discuss the similarities of galactic cosmic ray acceleration and ORBE/RCI, for the sake of brevity we constrain our discussion to ORBE/RCI alone.

Since the mechanism we propose is novel, this paper is necessarily qualitative, for there is not space to devote to both definitions and extensive derivations for each step. In the first section, we discuss plasma thermodynamics and the significance of trapping for effective acceleration. With an appreciation for entropic conditions, many inefficient acceleration mechanisms can be readily eliminated from consideration as the origin of ORBE/RCI. In the second section, we develop by analogy and simulation, the properties of three basic plasma traps: the dipole trap, the Fermi trap, and the quadrupole trap, and show by direct estimation, the relative importance of the quadrupole trap.

2. PLASMA THERMODYNAMICS

2.1. Entropy Considerations.

If one considers that a Q_1 heat engine operates by taking medium-entropy energy, Q_1, and converting it to low-entropy energy, W, plus a high-entropy energy, Q_2, in such a way that entropy either increases (irreversible, $S>0$) or remains the same (reversible, $S=0$), then one can view particle acceleration as a similar process whereby the accelerated population, by virtue of its higher temperature and/or decreased thermal spread, plays the part of low-entropy work. The advantage of this analogy is that it enables one to quantify accelerator efficiency, which is defined as the amount of work output for a given energy input, $\eta=W/Q_1$. From thermo-dynamics, we argue that if the work has zero entropy (say, all the accelerated particles are found in oppositely directed beams to conserve momentum, producing an infinite temperature), then the maximum efficiency of the acceleration is the well known $\eta=(T_2-T_1)/T_1$. As we reduce the infinite temperature of the work by degrading the oppositely directed beams into "wings" or "tails" on the distribution, the entropy of the work increases from zero.

Assuming that the ideal reversible, $S=0$, situation occurs when the entropy of the exhaust equals the entropy of the input, then this increase in the entropy of the work mimics an irreversible heat engine, with consequent decreasing efficiency.

We performed a trick to separate W from Q_2, arbitrarily separating energetic particles within the same distribution by employing Maxwell's demon to sort particles by their velocity. In our ideal example, we have extracted a subset of particles with high kinetic energy, but zero entropy, leaving behind a lower energy subset but with unchanged entropy. If $Q_2/T_2=Q_1/T_1$, even after extracting W, then conservation of energy implies $Q_2<Q_1$ and thus $T_2<T_1$. Nevertheless, any real "sorting" demon reduces entropy, if for no other reason than it reduces the number of states of the system, where $S=k \ln\Omega$, k being Boltzmann's constant, and Ω being the number of states of the system. In most discussions of Maxwell's demon, the conservation of entropy is found to occur by a subtle back-reaction of the output on the sorting mechanism, which eventually stops the sort. Likewise the accelerated particles can affect the input plasma to saturate the growth rate or efficiency.

This saturation is masked in neutral gases, where collisions operate so effectively that even with a resident "demon", all species are back in thermal equilibrium on the timescale of a few collision times, but as is well known in the rarefied, fully-ionized gasses of space, collisionless plasmas experience long-range Coulomb forces that cause a very slow return to equilibrium (*Collier93*). Therefore space plasmas are rarely observed to be Maxwellian, but often are found in metastable or non-equilibrium distributions, such as bi-Maxwellians and kappa-functions, which we interpret to be evidence of a saturation mechanism driving the distribution into a stationary state.

Now we have a conceptual problem, how can we describe the entropy of a "saturated" or "stationary" state that is not in thermo-dynamic equilibrium? We make an assumption that the ergodic theorem applies to these situations as long as the "relaxation time" for the stationary state reversion to a simple Maxwellian is much longer than the acceleration time or cycle time of the heat engine under consideration. That is, the stationary state is a "metastable" equilibrium, which permits us to treat acceleration mechanisms as a perturbation around this point, using all the techniques of equilibrium statistical mechanics.

Note that the absolute entropy need not be calculated, only the relative change in entropy. Since entropy is defined as a logarithm of the density of states, this relative change in entropy is insensitive to an additive constant in the number of states. That is, we need not concern ourselves with finding the number of states of 10^{14} particles per cubic centimeter plasma if this factor stays the same in both input and output distributions, so that we can bin our phase space into the largest partitions possible which remain unchanged in both the input and output. Therefore by application of our modified ergodic theorem, we assume that the closer a distribution is to such a metastable equilibrium (which factors out), the more likely it is to exist, or, the smaller the increase in entropy generated by an acceleration mechanism, the more likely the mechanism can explain the data. Thus the task of finding the origin of the ORBE/RCI, should be to identify the most probable acceleration mechanism that can be physically defended.

Krall and Trivelpiece [1986] discuss plasma thermo-dynamic limits to stability which can be fruitfully applied to the problem of evaluating the efficiency of any putative acceleration mechanism. The first step is to identify the stationary state. If such a distribution is highly unstable, say, a nonmonatonic bump-on-a-tail distribution, we can calculate a growth rate, or conversely a relaxation time for it to decay. To satisfy our modified ergodic theorem, such a relaxation time must be much longer than the putative acceleration mechanism under consideration. In the next section we divide the acceleration energy by this relaxation time, to define an average acceleration power for selected mechanisms. Our goal is simply to find the highest power acceleration mechanism that can explain the data.

A second use is to invert the concept of “stability”, and argue that acceleration proceeds the opposite direction, converting waves into energetic particles. Accelerating to an unstable distribution is attempting to swim upstream in the entropic current, and is highly unlikely. Therefore not only must the stationary state have a long relaxation time, but the final state must be relatively stable compared to the initial one.

2.2. Traps and Waves.

In addition to all the statistical heating mechanisms proposed for neutral gasses, magnetized plasmas are known to support both fluid (MHD) and kinetic waves, which add dozens of candidates for wave-particle acceleration. Winnowing the field down to the leading candidates should then be the first task of all analyses. Thus we develop a binary tree algorithm, divide and conquer.

The first division separates single-step and multi-step mechanisms, e.g., a shock passage heats the downstream fluid in a single step, just as a van de Graaf generator is a single-step accelerator, whereas Fermi acceleration or a cyclotron is a multi-step accelerator. From entropic considerations, we argue that multi-step acceleration is more efficient because it operates closer to equilibrium than a single-step process, the improvement being approximately the Boltzmann factor, or temperature difference of a single step taken in both mechanisms. This is not to say that single step accelerators are not useful, but from a global perspective, inefficient processes will only be apparent in the data when there is no other, more efficient process available to explain them. Only after exhausting all other more efficient mechanisms should we fall back on a single-step mechanism.

Next we distinguish between coherent and incoherent mechanisms. The difference may be thought of as that between a cyclotron and a synchrocyclotron. In the first case, the driving frequency is not necessarily in synchronization with the particle population, leading to acceleration inefficiencies, whereas in the latter case it is “tuned” to the particle packet. Consequently, the spatial scale of coherent acceleration is much larger than that of incoherent acceleration. Using Boltzmann's definition of entropy, we see that coherent systems have far fewer states of the system, or alternatively, the particles in the system are concentrated in far fewer states leading to much smaller entropy. A rough estimate of the effect is a factor of the scale size of the smallest scale length, perhaps a gyro-orbit, divided by the coherency length, raised to some power between 2 and 3.

Now we earlier argued that a reversible heat engine is to be preferred to an irreversible one, but a coherent system has overall lower entropy, should it also be preferred over an incoherent system? All other things being equal, yes, but thermodynamics warns us that we cannot reduce the entropy of the boundary condition without cost. Rather, if we draw a bigger box around the coherent boundary conditions, that system must increase in entropy too. That is, as long as there is some interaction between coherent waves and particles that is faster than the relaxation time of the particle spectrum, then we can augment phase space to treat the waves as a special type of particle and ask the question, “what is the efficiency of the heat engine that produces both the waves and the particles?” Note again, we need not consider the kinematics of wave generation, so long as we can calculate the entropy ratio of output to input.

Now the cost of coherent waves becomes clear. The input lacks both accelerated particles and coherent waves, whereas the output has both. Thus any mechanism which proposes coherent wave acceleration must include the entropy cost of providing for an even larger number of “accelerated pseudo-particles”. That is, acceleration by incoherent waves is to be preferred to coherent waves because they do not change the entropy bookkeeping of the heat engine, where the entropy cost difference can be estimated as the proportion of phase space occupied by the coherent waves in contrast to the incoherent waves.

The "cost" of coherency is similar to the calculation of the "Ockham factor" in Bayesian treatments of statistics (*Sivia96*). That is, if a data set can be fit either by a theory with two, or three adjustable parameters, the cost of one more variable can be calculated by estimating the volume of phase space introduced by the additional parameter. It is in essence the same entropic calculation, where we look at the range of available values for the additional parameter compared to the total volume introduced by the additional parameter.

"Wave" is merely an abstract term here, which can refer to any quantized energy source, dynamic or static. The reflection from a moving wall in Fermi acceleration, may be considered a "wave", where the continuous compression of approaching walls, Fermi-I, would be a "coherent" wave (each energizing step is in the same direction), and the motion of a particle between randomly moving walls, Fermi-II, would be an "incoherent" wave (diffusive energy gain). Thus Fermi-II is to be preferred over Fermi-I, all other things being equal.

The final division is the distinction between standing waves and travelling waves, or from the particle perspective, between a trap and free streaming. The difference is whether a given wave or particle interacts more than once in the acceleration process. If the waves and/or particles are spread uniformly through the entire thermodynamic volume, then the initial energy Q_1 must be much larger to produce the same output acceleration, W, and hence a reduction in the efficiency, η. Conversely, if we keep the energy constant by spreading the same number of waves/particles over a larger volume, then the dilution reduces the probability of an accelerating interaction and greatly increases the acceleration time, possibly invalidating our stationary state hypothesis. Thus it should be clear that the highest power heat engines are those that concentrate their work.

Therefore the most likely and highest power mechanisms are those that are multi-step, incoherent, and occur in a trap. For our application, three such traps have been considered: the dipole trap itself, the Fermi-trap, and the quadrupole cusp trap.

3. TRAP CHARACTERISTICS

In our thermodynamic heat engine analogy for acceleration, the entire argument depends on the approach to quasi-equilibrium. If any part of the process takes too long, then the entire method is invalidated. Therefore we must consider other temporal bottle necks beyond entropic considerations that can restrict the power of the output. For example a restricted source of particles might throttle the power, or a rapidly diminishing probability for multiple steps may produce too soft a spectrum to explain the data. At this point, we must leave behind generalizations and focus on the particulars of the traps as accelerators. In Table 1, we compare the Earth's dipole trap and its energization through radial diffusion (e.g., *Schulz74*) with both the Fermi-trap at the bow-shock (e.g., *Ellison90*) and the quadrupole trap found in the Earth's outer cusp (*Sheldon98, Sheldon00*). The quadrupolar acceleration mechanism is discussed later, but invokes the same SW compressive events used by others (e.g., *Mead64, Fälthammar65, Fillius67)* to produce dipole energization.

Table 1: Comparison of Three Traps

Feature	**Dipole**	**Fermi**	**Quadrupole**
1)Stochasticity	**poor**	moderate	good
$\tau_1 : \tau_2 : \tau_3$	10^{-3}:1:10^3	10^{-3}:10^3:10^4	10^{-1}:1:10^1
2)Process	**poor**	moderate	good
flow from …	Rim>Ctr	End>Side	Ctr>Rim
And exit is …	blocked	by diffusion	easy
3)Wave coupling	**poor**	moderate	good
varies w/Energy	inversely	constant	directly
4)Trapping for	moderate	**poor**	good
acceleration	traps	detraps	both
5)Diffusion and	**poor**	moderate	good
acceleration	needed	helpful	neutral
6)Adiabatic Heat	good	moderate	good
P.A.D.	2D oblate	1D prolate	2D oblate
7)Energy source	moderate	moderate	good
SW vs internal	compress	Alfvenic	both+intern

8a) electron E_{max}	good	**poor**	moderate
MeV @ Re	900 @ 10	1.8 @ 0.1	280 @ 3
8b) electron E_{min}	**Poor**	good	moderate
keV	< 45	2.5	~30
9a) Trap Volume	good	**poor**	moderate
log(m^3)	24	20	22
9b) Trap lifetime	good	**poor**	moderate
log(seconds)	> 13	4	lo:hi 9:5
9c)Trap Accel *t*	**poor**	good	moderate
log(seconds)	> 5.2	3.8	4.2
9d)Trap Power	good	**poor**	moderate
log(Watts)	< 8.3	6	7.3

As a chain is only as strong as its weakest link, so an acceleration process can be throttled by a single step. As the comparison shows, quadrupoles may be the most robust of the three processes. One could view it as a generalization of Fermi-acceleration to the other two spatial dimensions, or as an inside-out dipole trap. From this table, we see that the quadrupole is a very promising accelerator trap indeed, which while not conclusive proof for the origin of RCI/ORBE, is reason enough to discuss it in more detail.

3.1.1 *Stochasticity.* From Hamiltonian dynamics, the symmetries of a trap produce constants of the motion, adiabatic invariants such as (μ,J,L) for a dipole and a similar triplet for a quadrupole, while a Fermi trap lacks the third. Now $\mu \propto E_{\perp}$, while $J \propto E_{||}$, so in general, a particle cannot be accelerated without violating one or both of these invariants. Since the invariants are listed with increasing periods, an acceleration event that violates a particular invariant generally violates all invariants at longer periods as well, leaving shorter period invariants unchanged. This leads to the problem: Fermi acceleration violates the 2nd invariant, increasing $E_{||}$, but not $E_{\perp}$. Likewise, adiabatic compressions in both dipole and quadrupole traps violate the 3rd invariant, but have no effect on the more important 1st and 2nd invariants. In order to have an efficient accelerator, the energy must be redistributed rapidly among the invariants, so that particles do not detrap in the Fermi case, nor adiabatically return to their initial energy in the di/quadrupole case.

If we associate these invariants with a trajectory through phase space, then the conservation of the invariants maps to a closed curve in a Poincaré section, or an *n*-torus in the *n*-dimensional subset of 6D phase space (the Kolmogorov-Arnol'd-Moser theorem) (*Arnol'd64*). As fluctuations, scattering, and chaos cause these torii to "blur", Arnol'd argues that the torii can overlap and form a "web" that permits rapid stochastic transport through phase space, thereby redistributing the energy. Arnol'd shows that the formation of a web is greatly enhanced when the invariants have similar periods. The dipole trap, with three orders of magnitude separating the invariants, has no such stochastic transport, whereas the quadrupole trap is an ideal location.

3.1.2 Process. A second consideration is the flow of particles through the heat engine. The dipole has a large supply, the entire plasmasheet and geotail, but the acceleration process brings the output into the radiation belts close to the Earth, where they scatter in the upper atmosphere and are lost. All dipoles have this problem, that the exit is filled with the magnet that makes the dipole in the first place. Thus the dipole may be a bright source of energetic neutral atoms, but not energetic ions. The Fermi process is more dynamic, with the trap forming whenever the SW magnetic field is radial and vanishing abruptly as the field wanders. Thus the exit is not so much a flow as a sudden release. While the Fermi process isn't throttled, neither is it very continuous. In contrast the quadrupole process begins with plasma flowing into the center of the trap, where it is scattered and trapped. As it diffuses outward it is accelerated until it finally escapes at the rim. This process is limited not by the exit but by the supply of sufficient source particles at the center.

3.1.3. Coupling. The strength of the coupling between "waves" (power in disturbances to the trap), and the particles affects the power. In the dipole trap, the more energetic particles are also found much further in, and at much higher *B*-field strength, such that the strength of magnetic disturbances, $\Delta B/B$, as well as pitch angle scattering, decreases as energy increases, resulting in higher energy particles becoming more and more decoupled from the waves. In contrast, the Fermi accelerator imparts the same relative energy to all particles that rebound from a moving wall, independent of their energy. In absolute energy terms, the more energetic ones are actually favored. But the quadrupole trap, being an inside-out dipole trap, puts its most energetic particles at the periphery. So not only does a compression have all the beneficial characteristics of a Fermi trap, but the energetic particles are more likely to be in the largest $\Delta B/B$ region of the trap.

3.1.4. Diffusion. Diffusion being 2ndorder, is generally slower than 1st order direct acceleration, which is why the cosmic ray community has preferred Fermi-I at shocks to the slower Fermi-II, despite the lower entropy considerations of 2nd order methods. In addition, diffusion is most effective when the gradients are largest, so that diffusively dominated acceleration is a victim of its own success, becoming less efficient as it erases gradients. Thus the dependence and the rate of diffusion are both critical to understanding the efficiency or power of a proposed mechanism. Dipole acceleration depends completely upon diffusion to adiabatically energize, whose rate is a high power of radial distance, making it increasingly slower at higher energies. Diffusion is of secondary importance for Fermi acceleration, providing a way for energetic particles to escape the trap, or pitchangle scatter their $E_{||}$ into $E_{\perp}$ so as to achieve higher energization. But in a quadrupole, diffusion is almost irrelevant. The low field region at the center of the cusp acts as a built in scattering mechanism such that ordinary diffusion is of limited importance in redistributing the energy. Likewise, particles migrate outward in the trap under adiabatic, not diffusive forces, making the process flow independent of diffusion. This independence makes the quadrupole much faster than the dipole, and gives it a slight edge over Fermi in processing speed.

3.1.4. Adiabaticity. If we assume an adiabatic compression using a polytropic equation of state, $PV^{\gamma}=k$, where $\gamma=(m+2)/m$ is given by the number of degrees of freedom, m, then we can calculate how effectively a pressure pulse converts to work (acceleration).

$$dW=PdV=(k/V^{\gamma})dV=(k^{1/\gamma}/\gamma P^{-1/\gamma})dP$$

$$W=k/(1-\gamma)\,V^{1-\gamma} = PV/(1-\gamma) = k^{1/\gamma}/(1-\gamma)\,P^{\gamma-1/\gamma}$$

Setting k=1, for all systems, assuming a square-wave pressure pulse a factor n greater than the initial pressure, gives:

Table 2: Pressure Pulse Efficiency vs Trap Dimension

D	γ	W	1.01P	1.1P	10P	38P	100P
1	3	$.5k^{.3}P^{.6}$	.003	0.03	1.82	5.16	10.3
2	2	$1.k^{.5}P.^{5}$	.005	0.05	2.16	5.16	9.00
3	5/3	$1.5k^{.6}P^{.4}$	.006	0.06	2.27	4.90	7.96
1*	3	Normed	1.00	1.00	1.00	1.00	1.00
2*	2	Normed	1.50	1.49	1.19	1.00	0.87
3*	5/3	Normed	1.80	1.78	1.24	0.95	0.77

We see that for small pressure pulses (n=1.01), the 2-D quadrupole trap has a 50% greater acceleration efficiency than the 1-D Fermi trap, which holds true until the pressure pulse is roughly 38 times the initial pressure. Since small pulses are more common than large pulses, the quadrupole trap has the potential to be more efficient accelerator than the Fermi-trap, depending on k.

3.1.5. Energy sources. The energy source for the Fermi-trap comes from the B-field enhancement that reflects the streaming ions. Such an enhancement might come from Alfvén waves, or compressional waves in the SW, convecting toward the bow-shock. Very occasionally, it might be actual shock fronts propagating in front of a coronal mass ejection or magnetic cloud. Likewise, for the standard dipole compression, very similar disturbances in the SW are usually invoked. In terms of cross-sectional area presented to the solar wind, the dipole is largest, followed by the quadrupole, and lastly the Fermi-trap. In addition to these SW energy sources, the quadrupole can also absorb internal sources of waves, such as dipolarizations due to substorms. As *Hassam95* point out, the cusp has a very low Q-value, and is therefore a great absorber of wave power. While this source is available for the dipole as well, the $\Delta B/B$ is much smaller in the dipole than the quadrupole, reducing its importance there.

3.1.6. Energy cutoffs. The total energy in the accelerated spectra can be integrated over all energies, which for both Fermi and quadrupole acceleration, have power-law tails. Depending on the exact power-law, the cutoffs at both low and high energy have effect on the total. In addition, data constrains the models to very precise cutoff energies. The Fermi trap has an upper energy cutoff that occurs when the gyroradius of a particle exceeds the ~1° requirement for a quasi-parallel shock, whereas the cutoff for the dipole/quadrupole traps occur when the gyroradius is the same radius as the trap. Using 10 nT for the Fermi trap, and 50 nT for the dipole/quadrupole trap outer boundary, we estimate the maximum cutoff energy given 10 Re, 0.1 Re, and 3 Re for the respective radii of the traps.

Likewise, the low-energy cutoff can occur when $\boldsymbol{E} \times \boldsymbol{B}$ drift is comparable or greater than the trapping ∇B drift. Using the same estimate for the radius of the trap as above, and estimating the voltage from $V = r * \boldsymbol{v} \times \boldsymbol{B}$ of the SW, we get 2.5 keV for the Fermi trap, 360 keV for the quadrupole, and 1.2 MeV for the dipole. Clearly this is an overestimate, perhaps because both the dipole and quadrupole traps have boundary layers that short out much of the potential that develops from $\boldsymbol{v} \times \boldsymbol{B}$. Using satellite electric field probes, we can put more realistic limits of $E_{min} < 45$ keV for the dipole and perhaps $E_{min} \sim 30$ keV for the quadrupole.

Now the Fermi trap lower limit is just above the thermal energy of the SW, so that much of the SW particle distribution is available for acceleration, whereas both the dipole and the quadrupole have lower cutoffs greatly above the thermalized SW energy, which can starve the input of both these traps. This is the essential difference between “low” and “high” quadrupole states, where we propose the high state has modified cutoffs due to topological changes in the trap. That is, a CDC increases the radial magnetic gradient, which strengthens ∇B-drift, and effectively lowers E_{min}, while simultaneously raising E_{max}. In doing so, it increases the average power, produces CEP particles, and taps into the high fluxes available at lower energy.

3.1.7. Power. The last four entries are an attempt to estimate the average power of the proposed mechanisms using constant SW input. The trap volume is estimated for a dipole of radius 10 Re, a Fermi trap with a 1° wide region of a bowshock with a 12 Re radius of curvature extending 100 Re upstream, and a quadrupole of 3 Re radius and approximately 3 Re depth. The trap lifetime is estimated for a dipole to be the 1 million year flipping of the Earth's internal dipole field, for a Fermi-trap a 3-hour persistence for a particular vector direction of the magnetic field, and for a quadrupole trap, two separate persistence times. The first given by the dipole + solar wind persistence time, the second given by the proposed “high” meta-stable state of the quadrupole (estimated from risetimes of ORBE during high-speed solar wind conditions).

In calculating the acceleration time, it is not just the time for which the trap exists, but the time to accelerate a particle to the appropriate cutoff energy. Or conversely, if the trap exists for insufficient time, the energy cutoff will be correspondingly lower. We give an estimate for the time to go from 1 keV to 1 MeV in all three traps. Estimating this for the dipole trap is difficult, since one pass from L=10 plasmasheet to L=5 ORBE is insufficient to explain the spectrum, and no theory of multipass (e.g., Nishida recirculation) is currently accepted or understood. Nevertheless, we optimistically estimate that 4 circulations of 1 day each can provide the energy. For the Fermi trap, we estimate the time required to bounce 100 Re between barriers, receiving a 400 km/s kick at one end, or 32 kicks or, $t = (800{,}000 \text{ km}/400 \text{ km/s}) \sum 1/n = 2000(4.05) = 8000$s. For a quadrupole we assume a pulse of 30% increase in pressure, which from Table 2 gives a 14% increase in energy, or 53 kicks, which if occurring every 8 minutes (a typical peak in a power-spectrum of solar wind pressure pulses), integrates to 25,000 sec.

Clearly the dipole trap exists longer than the (uncertain) acceleration time, so power is limited by the acceleration time. In contrast, the Fermi trap has an acceleration time comparable to the trap lifetime, which means the power is limited mainly by the lifetime of the trap. The quadrupole trap is a little less clear. The “low” state exists much longer than the acceleration time, whereas the “high” state is again comparable. Thus in the high state, the power may again be limited by the trap lifetime.

The acceleration power is proportional to the energy density, ε, and volume, V, divided by the time, t, given by, $P_a = \varepsilon V/t$. We don't know the energy density well, since it can depend on waves as well as particles, but it should be comparable to some fraction of the solar wind energy density that can be extracted by the trap, e.g., 10% of the SW kinetic energy. So assuming the energy density has a constant value of $\varepsilon = 10^{-10}$ J/m^3 for all, we arrive at a power estimate of the three traps.

4. CONCLUSIONS.

We have made qualitative thermodynamic arguments for the superiority of traps in accelerating particles. Greater rigor could be obtained by evaluating actual distributions without any change in the argument, but at the risk of losing the forest for the trees. We then consider two well-known traps, and the lesser known quadrupole trap. By estimating the average power of the three traps from many perspectives, we showed that the quadrupolar trap has the potential to outperform the others, both in the magnetosphere and in astrophysical magnetospheres. In a subsequent paper, we refine the model for the Earth's cusp, showing the effect of CDC entrained plasma on the topology and energy cutoffs, which may account for the “low” (without CDC) and “high” (with CDC) states of the quadrupole accelerator.

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