Proposal Cover Sheet

NRA 96-OSS-09
Sun-Earth Connection
Supporting Research and Technology,
Suborbital, Guest Investigator,
and Education programs

Program Element:	ISTP Guest Investigator Program
For All Program Elements:	Proposal Includes Education Outreach Supplement? No
Technique/Research Area:	Theory and Data Analysis/ Magnetosphere
Title of Investigation:	Remote Sensing of Magnetospheric Structure Using (U,B,K) Analysis of ISTP Particle and Field Data
Principal Investigator:	Dr. Elden C. Whipple, Affiliate Professor Geophysics Department, Box 351650 University of Washington Seattle, WA 98195 (206) 616-5796, whipple@geophys.washington.edu Signature:_______________________
Co-Investigators:	Professor George K. Parks, same address as PI

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Authorizing Signature: _______________________

Budget Summary:	1st Year	2nd Year	Total
UW	$32,588	32,588	64,974
BU	$42,025	43,706	85,731
GSFC	$ 7,900	7,900	15,800
Total:	$82,513	84,194	166,707

Proposal Summary

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Title of Investigation:	Remote Sensing of Magnetospheric Structure Using (U, B, K) Analysis of ISTP Particle and Field Data
Principal Investigator/Institution	Dr. Elden C. Whipple, University of Washington

Summary of Proposed Investigation:

(a) Magnetospheric measurements with the sophisticated instruments on the ISTP spacecraft exhibit complex particle and field structures such as co-existing hot and cold plasmas, different flows for different species of particles, boundaries in physical space between different regimes and in velocity space between different kinds of plasmas. These features result from and are clues to the variety of processes affecting magnetospheric structure and require a similarly sophisticated analysis to gain quantitative understanding.

We propose to analyze ISTP particle and field data in collaboration with other ISTP investigators by using the (U, B, K) coordinate system (U = electric potential, B = magnetic field magnitude, K = modified longitudinal invariant) which has the property that particle mirror points follow straight lines in this system. Particle trajectories and their relation to magnetospheric regions are easily visualized which gives this analysis procedure a remote sensing capability to use particles as probes of regions through which they have travelled. In addition, computations involve mainly the 3D task of mapping between coordinate systems rather than the 6D task of following particles. We give examples of how this approach has solved observational puzzles and provided insight into magnetospheric processes. We propose work in three areas:

(1) Treatment of specific scientific problems: We propose to study ISTP observations of phenomena caused by remote processes, such as the deep minima observed frequently in ion spectrograms, and particle "nose events" and "zippers". These features are signatures of processes occurring at remote distances from their observation and we believe that (U, B, K) analysis can be especially fruitful in extracting the physics of these processes. We will use ISTP measurements of electric and magnetic fields together with particle data to improve our knowledge and construct better models of the most poorly known magnetospheric quantity, its electric field. The (U, B, K) system provides a way for extending such models beyond the measuring point to remote regions.

(2) Preparation and distribution of of data analysis tools: We will develop algorithms for mapping between the ordinary and (U, B, K) systems with the use of different models of magnetic and electric fields, and will provide algorithms for identifying boundaries, especially the boundary between adiabatic acceleration and decceleration regions which is crucial in determining particle access. We will study how field models can be used to identify particle trajectories that connect pairs of spacecraft and how comparison of particle data from different spacecraft can in turn be used to improve the models.

(3) Firming up the theoretical foundations and extending the applicability of this approach: We will examine how to extend the analysis to include particle drift kinetic energies, relativistic velocities, time dependence, scattering and diffusion, and parallel electric fields. We will examine how a Hamiltonian formulation of the particle mirror point motion might be useful for obtaining a mirror point phase space density to be used for providing moments and perhaps new moment equations.

(b) New Task

(c) Dr. Whipple is a Co-Investigator on the Hydra instrument on Polar. Hydra data analysis tasks done by Dr. Whipple will be funded from the Hydra project. Dr. Parks is a PI for UVI on Polar and a Co-I for the 3D/Plasma instrument on Wind. Dr. Sheldon is working with Dr. Spence who is a Co-I on Polar CEPPAD. CEPPAD tasks will be funded from that project. The proposers will collaborate with other ISTP investigators on any task where their data is requested. The theoretical work will be carried out primarily by Drs Whipple, Northrop and Sheldon. All the investigators will participate in data analysis. Drs Parks and Spence are contributing their time at no cost.

(d) The proposed research is particularly relevant to the ISTP goal of using multiple spacecraft data for correlative science on a global scale.

1 Introduction

1.1 Background

A key feature of the Earth's magnetospheric plasma is that both the solar wind and the ionosphere are sources. As a result of the long mean free path for binary collisions and the relatively weak Coulomb and wave-particle interactions, the hot plasma from the solar wind and the cooler ionospheric plasma tend to retain their distinct identities for considerable time. Thus, observations in the magnetosphere reveal a complex and dynamic plasma, with hot plasmas coexisting with cold plasmas. These different populations in general occupy their own niche in phase space, and each experiences its own distinct scattering, energization, and loss processes. In addition, ions follow different paths than electrons through the effects of the magnetic gradient and curvature drifts. For example, ions with certain specific energies coming in towards the earth from the magnetospheric tail can penetrate to low altitudes as the drift changes from westward to eastward, whereas this is not true for electrons.

McIlwain and Whipple (1986) surveyed the dynamic behavior of plasmas near geosynchronous orbit and found the plasma behavior so variable that it was difficult to characterize. Two predominant features were sudden injections of hot plasma and the presence of marked magnetic field-aligned structure. Ions and electrons frequently displayed different field-aligned structure which changed with energy. Other phenomena were ion clusters bouncing between hemispheres, separate patches of low-energy plasma, and various kinds of evidence for plasma heating. *Frank and Paterson* (1994) surveyed Geotail electron and ion data in the distant magnetotail and found that bulk flow speeds were different for electrons compared to ions, and that the disparity at times was due to the presence of two co-existing ion velocity distributions (one hot and one cold). The boundaries between the different plasmas occur in both physical space (e.g. at the plasmapause and the magnetopause, etc.) and in velocity space (e.g. at edges of loss cones and inverted-V events, etc.). Other examples of boundaries are Alfvén layers which separate different drift trajectories, the inner edge of the plasma sheet, and the boundary between the lobes and the mantle, etc.

Although MHD formulations can frequently provide a reasonable description of the "average" behavior of the magnetospheric plasma, e.g. (*Siscoe et al.*, 1994, *Frank*, 1995), nevertheless it remains true that distinct features in measured particle distributions provide clues to the source of those particles and to the processes that have affected them during their transport from their source to the point of measurement. Thus a kinetic formulation for the transport of individual particles can be an extremely valuable tool for unravelling some of the complexities of magnetospheric processes. In this spirit, Ashour-Abdalla and co-workers have pioneered the "large scale kinetic" approach to studying the transport of individual groups of particles in a global electromagnetic environment and have been able to show that both the ionosphere and the solar wind are at times sources for parts of the plasma population measured at a given point in the magnetotail (*Ashour-Abdalla*, 1996). Since present-day plasma instruments have such remarkable resolution in time, energy, and direction, a corresponding sophisticated theoretical treatment of the particles in the plasma is required to arrive at an understanding of the complex features that have been revealed in the measured particle velocity distributions. A major difficulty with kinetic treatments has been the extreme demand on computational power required to adequately follow large numbers of particles.

1.2 The (U, B, K) Formulation for Plasma Transport

McIlwain (1972) used ATS-5 particle data to obtain a model for the magnetospheric electric field. He pointed out that mapping the equatorial plane into a new coordinate system given by the electric potential and the magnetic field intensity was particularly valuable, since the drift trajectories of locally mirroring particles (i.e. 90° pitch-angle) were given by straight lines. He was able to use this analysis to "remotely sense" the electric field over a large volume of the magnetosphere even though the spacecraft remained in synchronous orbit. This simple form for the drift paths of the particles in the new coordinate system resulted from the assumed adiabatic invariance of the magnetic moment (μ), so that the total energy (E) could be expressed in terms of the electric potential (U) and the magnetic field magnitude (B) as:

$$E = eU + \mu B \tag{1}$$

In the (U, B) plane, drift trajectories were linear functions of U and B (i. e. straight lines), with ions having a negative slope and electrons a positive slope. McIlwain was able to explain features of particle data, such as the occurrence of deep minima in the ion fluxes at particular energies, by showing that these trajectories were for ions penetrating deep in the magnetosphere towards the earth where they were depleted by loss processes.

Whipple (1978) extended this work to all pitch angles by invoking the modified longitudinal invariant (K) which depends only on position in the magnetosphere through the magnetic field topology and not on any particle property as long as magnetic field lines are equipotentials e.g. (*Roederer*, 1970):

$$K(\mathbf{r}) = \oint \sqrt{B(\mathbf{r}) - B(s)}ds \tag{2}$$

On a surface of constant K, as long as the second invariant remains preserved, the particles mirror-point trajectory can be followed in the (U, B) plane and again follows a straight line path. It was further shown that a generalized scalar velocity function (W) can be defined that depends only on the electric and magnetic field configurations such that the mirror-point velocities in the new coordinates are given by:

$$dU/dt = -(\mu/e)W \tag{3}$$

$$dB/dt = W \tag{4}$$

$$dK/dt = 0 \tag{5}$$

$$W = \frac{[\nabla U \times \nabla B] \cdot \nabla K}{\mathbf{B} \cdot \nabla K} \tag{6}$$

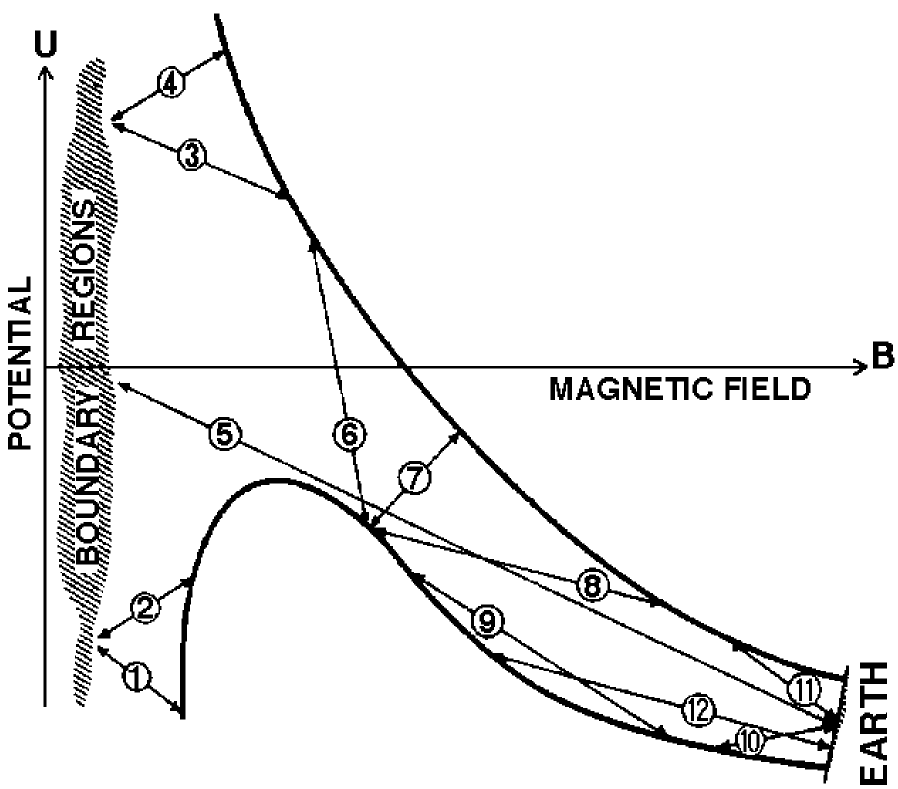

Figure 1: UB space particle trajectories in the K=0 plane: the W=0 bounding curves are turning points where the particles cross between the tail and front of the magnetosphere. Thus both regions can be represented on the same figure, with the sense of motion determining the region. Trajectores 1–5 connect to source regions at low magnetic field, 6–9 are closed trajectories, 10–12 are precipitating or coming from the ionosphere in the drift source cone. Electrons have positive slope, ions negative slope. Cold plasma has vanishing slope.

The function W has the additional property that it divides the magnetosphere into dynamo regions ($W < 0$) and adiabatic acceleration regions ($W > 0$), with the boundaries between these regions (where $W = 0$) forming "turning-points" where the velocity components vanish, separating outward moving (towards smaller B) from inward moving particles. The occurrence of forbidden zones and the distinction betweeen various types of particle trajectories are determined by the shape of these boundaries and are easily visualized in (U,B,K) space. Figure 1 illustrates the types of particle trajectories that are easily distinguished in the (U, B) plane. The $W = 0$ boundaries are especially useful in determining the nature of allowed drift paths since they act as barriers to particle access. We point out later that W can be determined locally quite well from spacecraft field measurements. The reader is referred to *Whipple* (1978) for detailed derivations and discussions of the (U, B, K) formalism.

The power of the (U, B, K) approach lies in the extreme simplicity of the particle trajectories and in the easy visualization of these trajectories in their relation to various magnetospheric regions. The computational tasks are now almost entirely involved with the mapping back and forth between a conventional coordinate system and the (U, B, K) system, which is a much easier task (a three-dimensional transformation) than that of following a large number of particle trajectories in a Cartesian system (a six-dimensional problem).

We believe that the (U, B, K) coordinate system provides a unique tool that could prove of tremendous value for the analysis of ISTP spacecraft data, especially in the present environment with the great emphasis on data coordination from multi-spacecraft observations. The greatest contribution that use of this analysis technique could bring to the ISTP program is its ability to use particle and field data to provide a "remote sensing" capability. It has always been known that particles sensed at one spacecraft location have arrived there by travelling a great variety of lengthy and tortuous paths through different magnetospheric regions, but there has been no easy-to-use analysis method for making use of these particles as probes bearing information about the regions through which they have travelled. The (U, B, K) system provides such a capability.

The PI of this proposal is a Co-Investigator on the Hydra plasma instrument on the Polar spacecraft and has in that capacity been asked to consider how the (U, B, K) system could be applied to the analysis of Hydra data. However, there are a number of limitations in using the (U, B, K) system at present. For example, it assumes a steady state for magnetic and electric fields, its formulation is non-relativistic, and it assumes no parallel electric fields. The application of this approach needs to be extended to wider regimes in order to overcome these limitations. These considerations go beyond its application to the analysis of data from one instrument. Then there is the practical need to make the use of this approach "friendly" for potential users.

The tasks we propose to carry out fall into three categories: (1) Scientific Problems to be Addressed (with examples of how this approach has facilitated insight into similar problems in past data), (2) Preparation and Distribution of Tools for Data Analysis, and (3) Deepening the Basis and Broadening the Applicability of the (U, B, K) Approach.

2 Proposed Tasks

2.1 Science Problems to be Addressed

We begin our discussion of proposed tasks by addressing a number of science problems that we believe are especially open to analysis by use of the (U, B, K) approach. With ISTP there is a rich data set to analyze with this new tool.

2.1.1 Analysis of Phenomena Caused by Remote Processes

Deep Minima Observed in Ion Spectrograms *McIlwain* (1972) showed that deep minima frequently observed in ion spectrograms could be explained through scattering or charge exchange deep within the plasmasphere. The minima occurs at energies corresponding to trajectories which penetrate to low altitudes, deep in the potential well in the (U, B) plane (Fig. 2). It is no coincidence that this feature is often associated with the "nose protons" discussed earlier. Both phenomena occur because of the unique set of ion trajectories that lie between eastward and westward propagating paths. "Nose" events correspond to particles that have high fluxes compared to surrounding trajectories that are depleted, whereas minima correspond to trajectories where particles have been lost. Both of these features are signatures of remote processes. We propose to study how to use these features to gain information about magnetospheric structure and processes.

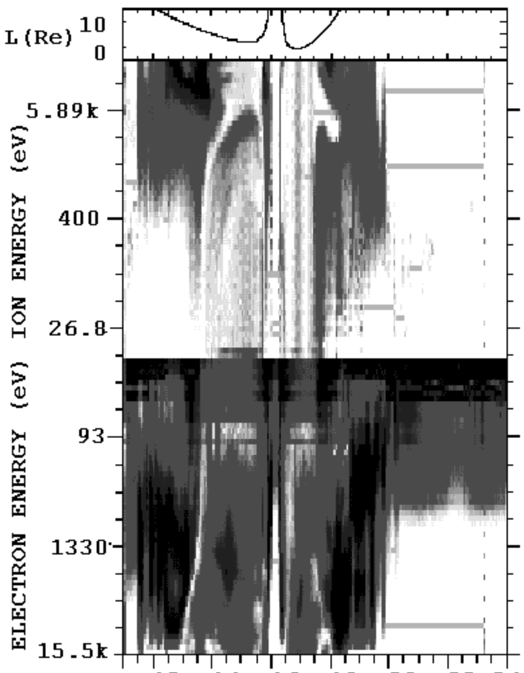

Figure 2: POLAR/HYDRA ion and electron spectrograms showing a deep minima traversing both species at 1300 UT on June 20, 1996.

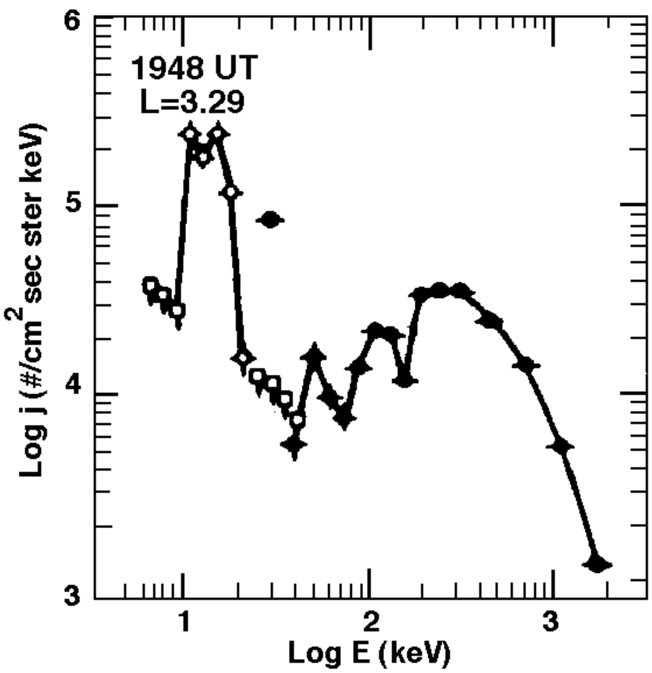

Figure 3: Williams and Frank (84) mystery peak. Open symbols are LEPEDEA, filled circles are MEPI.

Example from Past Data: "Williams and Frank" peak When *Williams and Frank* (1984) analyzed the lowest energy channels (~20keV) of the ISEE MEPI instrument, they found a large peak (only a few channels wide) in certain regions of the inner magnetosphere, L~3, at dusk (Figure 3). This observation was so unexpected they included the upper energy channels of the LEPEDEA instrument to show that the effect was not instrumental. The pitch angle distribution evolved in a way completely contrary to usual spectra or background. The data could not be dismissed as a fluke, since the peak persisted over several months, whenever the ISEE orbit crossed this location. Despite two publications, the only theory advanced was a localized ion-cyclotron acceleration that was very selective in energy and spatial dependence, but did not address the pitch angle evolution.

Sheldon (1994c) reanalyzed the data within the UBK framework and found that this peak could be explained as the orbit of the most deeply penetrating plasmasheet population, sandwiched between closed orbit trajectories on both sides (Figure 4). During quiet periods, the closed orbits are depleted of plasma, making this trajectory stand out as a peak. This analysis predicted not only the correct plasma density, but it predicted the correct pitch angle dependence as well. Moreover, it accomplished this analysis with no adjustable parameters on either fields or particles. Perhaps one reason that this phenomenon was not revealed with particle tracing codes was that the volume of phase space for this deeply penetrating component was vanishingly small, requiring resolution of many thousands of particle orbits. Regardless of the reason, UBK transformations were able to explain a previously unexplained mystery, another indication of the power of the technique.

"Zippers" Observed in Loss Cone Structures

ISEE data – zippers *Fennell et al.* (1981) found so called "zipper" distributions in ISEE data in which a low energy (<2 keV) field-aligned, prolate PAD transformed into a high energy (>10keV) trapped, oblate PAD. No convincing theory was advanced that could explain all the features of the data, though tantalizing discoveries were made that the prolate flux was enhanced in ionospheric species. We believe that

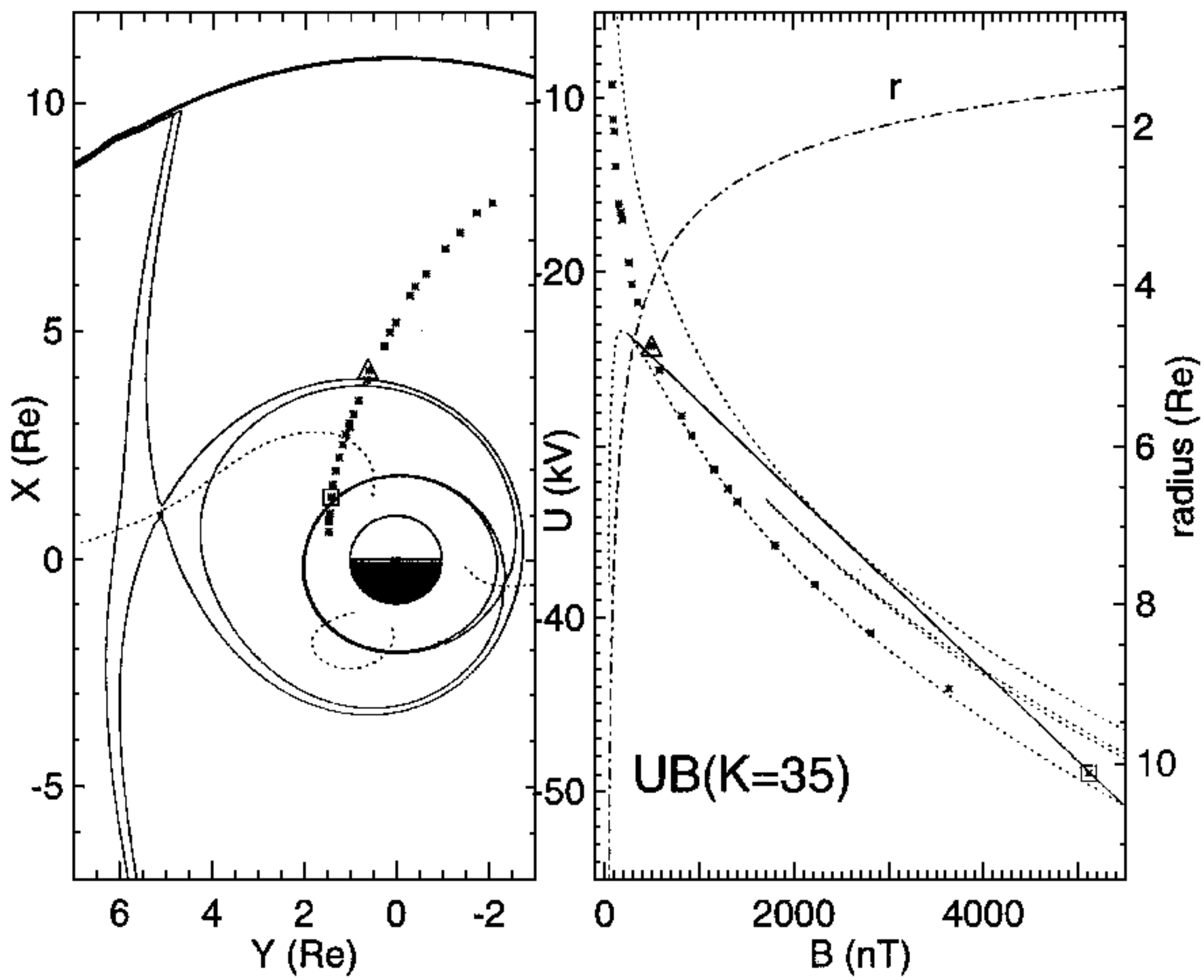

Figure 4: The deepest penetrating ion trajectory is bracketted by solid lines in the left figure, ISEE track shown as crosses, intersections marked with triangle and square.

UBK analysis of the K-dependence of accessible drift orbits will show that parallel electric fields combined with a cross-tail electric field can describe all the features of the data.

Polar data on 4/15 On April 15, POLAR measured what is possibly the most intense zipper distribution ever observed (Fig. 2.1.1). Prolate distributions continue up to 30 or 40 keV. Very large double layers would have to be invoked to account for these ions. We believe that there may be a direct causal link between the "nose" event population (seen >60keV) and the presence of parallel electric fields. *Whipple* (1977) has shown that these parallel electric fields may be produced by peculiar PADs, which in turn may be produced by intense convection electric fields, as evidenced by the "nose" events.

Example from Past Data: Injection "Nose Events" When the $E > 30$ keV, L=3-5 ion data from Explorer 45 was displayed as an energy-time spectrogram, features known as "nose events" were observed during storms or substorms, *Smith and Hoffman* (1974). In these events, a 30 keV ion enhancement occurs at the spacecraft that slowly broadens in energy until the enhancement covers the entire energy range. The name is thus derived from the characteristic shape of the event in these coordinates. Now in order for a 30 keV ion to arrive first in a dispersed injection, the energy dispersion had to unexpectedly reverse sign at 30 keV.

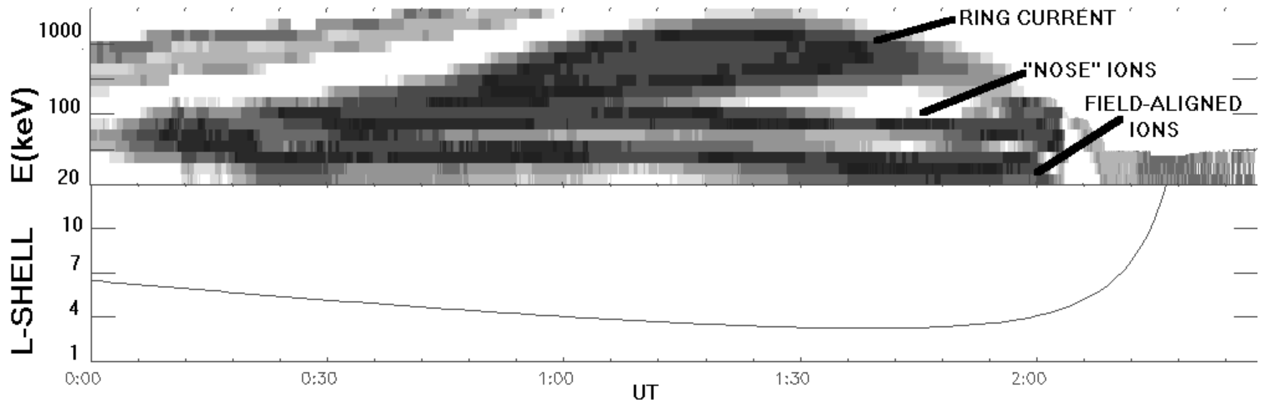

Figure 5: POLAR/IPS ion spectrogram from April 15, 1996 showing two distinct energy bands, the higher energy with oblate PADs, the lower with prolate PADs (field-aligned). See http://buspace.bu.edu/EPG/rsheldon/.

The analysis required a rather detailed simulation of the entire storm using particle tracing (Langrangian methods) to trace many ions through the magnetosphere and show the characteristic dispersion. This entire analysis was duplicated by *Whipple* (1978) with two straight lines drawn in (U, B, K) space. (Figure 7).

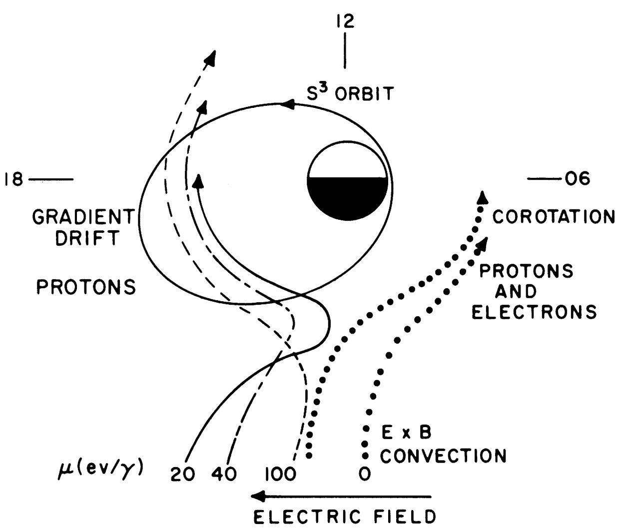

Figure 6: "Nose" events as explained by *Smith and Hoffman* (1974) showing deepest penetration of 20 eV/γ ions.

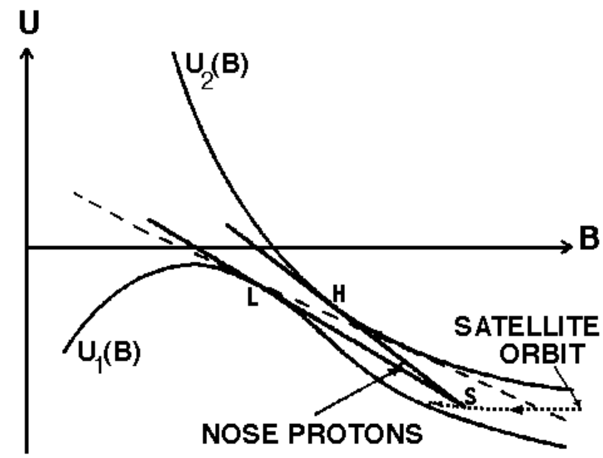

Figure 7: "Nose" events as explained by *Whipple* (1978). Ion trajectories H and L mark energy limits on open trajectories available to spacecraft at point S. Dashed line marks the last open trajectory.

2.1.2 Scattering and Diffusion

The static Hamiltonian formulation of the (U, B, K) system can be easily generalized to small perturbations and a diffusion coefficient can be defined (*Sheldon and Eastman*, 1996) in energy space. The major advantage of this approach is the generality of the derivation–it is equally valid for low and high energy ions of all pitch-angles in real or ideal field geometries. Application of the Jacobian can convert this diffusion coefficient into the more well-known diffusion operator, D_{LL}. The entire derivation requires no more than 5 equations and compares favorably with the 10 page derivation of *Nakada and Mead* (1965) presented 30 years earlier. More importantly, it generalizes to arbitrary magnetic and electric field models, and improves the diffusion coefficients for both the inner and outer radiation belts. However, the method is not yet generalized to

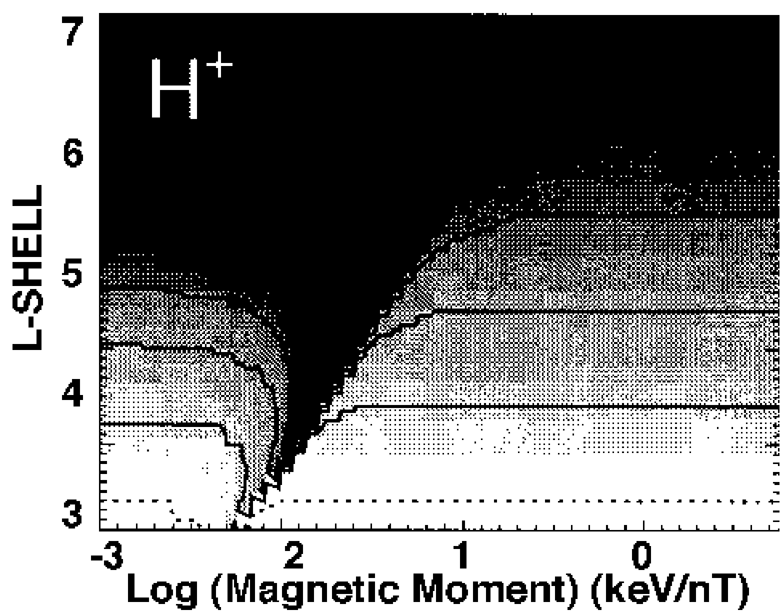

Figure 8: Radial penetration of trapped and open ion orbits averaged over all LT.

diffusion in the first two adiabatic invariants, μ and J. We hope that once this is done, we will have generalized the fundamentals of radiation belt dynamics as epitomized in *Schulz and Lanzerotti* (1974) book to the more realistic magnetic and electric field models available today, thus providing a significant improvement to radiation belt diffusion models.

Example from Past Data: Valid Diffusion Domain The higher energy ions (>100keV) in the magnetosphere are efficiently transported across L-shells via diffusion. But the standard diffusion model presupposes that the ions are on circular drift trajectories that remain at a constant radial distance from the earth. In his analysis of the ring current diffusion model (*Sheldon*, 1994b), UBK coordinates were used to find the range of validity of the diffusion model, where this assumption was valid. In this study, the average radial spread of all drift paths averaged over all local times had to be calculated (Figure 8). This would have required many days of CPU time using a traditional Langrangian particle tracer, but was reduced to a simple geometric algorithm in UBK space, using only minutes of CPU time. Suprisingly, this diffusion boundary occurred at higher energies than expected, bringing into question the validity of other ring current studies.

2.1.3 Using Spacecraft Measurements of E and B to Improve Electric Field Models

The largest uncertainty in mapping particle drifts is identifying the appropriate electric field model to use. We propose to invert the mapping tool to calculate the electric field from particle measurements. This has a number of advantages over competing methods that rely on simplified magnetic and/or electric field models in that arbitrary (such as McIlwain E5W) fields can be used as input or output.

The determination of a global electrostatic potential from *in situ* measurements of the gradients is not trivial. However calculating the energy Jacobian, W, described above has a number of advantages. One is that it allows a local measurement of the "particle energy flux" and the identification of regions in which particles are being adiabatically compressed or decompressed. This meets one of the prime goals of ISTP, namely the evaluation of energy flow in global geospace. A second advantage is that one can derive the electrostatic potential with fewer assumptions, and test them with the energy flux. In essence we can develop a tool that provides two independent methods of calculating the global electrostatic potential, and therefore can minimize the errors.

The proposed procedure is to use ISTP electric and magnetic field measurements together with the (U, B, K) system to improve models of the electric field. The idea here is to get the velocity function W, as given by eq. (4), directly from local measurements. Note that W depends directly on the electric field and on the gradient of the magnitude of the magnetic field. The latter is not directly measured but for a large part of the magnetosphere the magnetic field is known well enough that a local measurement of the vector B together with a good model for B, when combined with a local measurement of E, can provide the local value of W. The significance of the velocity function W is that its topology, and especially the shape of the $W = 0$ curves, determine the nature of the drift paths that have access to the spacecraft. A map of W obtained over a number of spacecraft orbits during various conditions can therefore serve as a test for correcting and improving models of the electric field by comparing with particle data as seen from different spacecraft locations.

There is an analogy here with how optical data can be used to obtain information on the medium through which light propagates. In the (U, B) plane, particle trajectories are straight lines, but if the electric and magnetic fields are not well known, then the mapping will yield particle trajectories which will be inconsistent with the particle data obtained from the spacecraft. For a pair of spacecraft, depending on their locations, there could be a set of particles which can be seen by both spacecraft (see section 2.2.3). (U, B, K) analysis can identify the range of particle energies and pitch angles for each spacecraft which defines this mutually observable set of particles, if indeed the electric and magnetic field models are reasonably accurate. If the models are not good, then the "views" of the particles from the spacecraft will be distorted. By this we mean that the distribution functions for the common set of energies and pitch angles as seen by the two spacecraft will be inconsistent with each other in some way. This distortion can in principle be used to correct the models through the kind of inversion technique that is used in optical studies, where refraction, scattering, absorption, etc. can be used to obtain information on the medium. In our case the "medium" is the electric and magnetic field configuration together with other effects such as pitch angle scattering, etc.

Example from Past Data: Electric field Extraction Since nose events mark the boundary between open and closed ion drift paths, these ions are tracers of the Alfvén boundary where the lowest energy Alfvén boundary is the plasmapause. McIlwain first used the (U, B) coordinate system to extract the electric fields from ATS-6 particle data. Another analytic inversion technique was used by *Maynard and Chen* (1975) to extract the appropriate coefficients for a Volland-Stern (*Volland*, 1973, *Stern*, 1973) electric field from the plasmapause. This same analytic approach was also used to extract electric fields from the CRRES data set. In contrast, (*Sheldon*, 1994a) used the original UBK technique applied to the AMPTE/CCE data set to automate the process of extracting electric fields for an arbitrary E and B-field model. While no faster than the analytic inversion technique, it had the advantage of being completely general for any (stationary) model E and B field. He was able to show that these Alfvén boundaries occured primarily in the region of dipolar magnetic fields, which are nearly invariant for all magnetic field models tested, so that the technique was only sensitive to details of the electric field model.

Example from Past Data: Inverse Electron Dispersion The electron Alfvén boundary produces an unusual dispersion signature at the border of the plasmasheet where low energy electrons were observed closer to the Earth than higher energy electrons. Although qualitatively understood by invoking the energy dependence of ∇B drift, the quantitative understanding of this dispersion was lacking. *Horwitz* (1984) used UBK analysis to not only explain this effect quantitatively, but to estimate the required cross-tail electric field needed to duplicate the observations. In essence, he was able to use the UBK transformation to extract the entire cross-tail potential from energetic particle measurements.

2.1.4 Use of Quasi-Neutrality to Obtain the Convection Electric Field

The current view of the magnetospheric electric field is that the solar wind $v \times B$ field is imposed across the magnetosphere and that the magnetospheric plasma then reacts to "shield" the magnetosphere such that the imposed field is modified through the motions of the magnetospheric ions and electrons and the concomitant production of space charge. However, there does not seem to have been any attempt to use this scenario to actually calculate the effects of this shielding process from first principles. We believe that the (U, B, K) system may be able to provide an approach to doing this.

The idea that will be used is that in each constant K surface, the $W = 0$ curves in the (U, B) plane control the access of particles to each point in that plane. Sources for the plasma can be the solar wind (at low values for B) or the ionosphere (at high values for B: see Figure 1). By assuming appropriate distribution functions at the source regions and using Liouville's theorem (at least for a first attempt at this problem), particle densities can be calculated at any point, using the (U, B, K) system in which trajectory calculations are trivial. Ions and electrons have access along different drift paths so that in general there could be significant space charge generated. However the fields will react to minimize the space charge, and we propose to calculate this reaction by imposing the "quasi-neutrality" condition (setting the space charge equal to zero; see *Whipple* (1977)) which then imposes conditions on the W = 0 curves and therefore on the electric field, assuming that the magnetic field is known. Appropriate boundary conditions would be that the electric field approaches the solar wind field outside the magnetosphere and the co-rotation field plus ionospheric convection near the earth. We propose to explore this approach towards understanding how the magnetospheric electric field is generated.

2.2 Tools for Data Analysis

The (U, B, K) approach provides a powerful tool for visualizing particle trajectories and how they connect different regions of the magnetosphere. At the same time the technique provides the capability of providing quantitative results for a variety of magnetospheric process, as illustrated above. Yet the technique has not been widely used, partly no doubt because of the difficulty of becoming acquainted with a new way of looking at the magnetosphere. One of the important goals that we have is to develop and provide "user friendly" tools which will make it easy to use this tool, to visualize the particle trajectories in both (U, B, K) space and in real physical space. We also aim to provide software packages for the mapping back and forth between coordinate systems and for related calculations that are at the heart of the technique. We will work with the various particle and field investigator teams on the ISTP spacecraft to familiarize them with the use of the (U, B, K) analysis technique and to study how to best adapt these tools for analysis of their data.

In earlier work we developed tools for carrying out the mapping between arbitrary field models and UBK space (*Sheldon and Gaffey*, 1993, *Sheldon and Eastman*, 1996) (see Figure 4). These tools have provided the basis for the analyses described above. But they need to be made more flexible and more general. We propose to develop a simplified graphical user interface (GUI) for these tools as well as making them more general as described in the previous section.

2.2.1 Magnetic and Electric Field Models

There are several magnetic field models which are in use at the present time, each of which has advantages and disadvantages since each is based on particular data sets and is therefore best in a particular region of space. In addition, there are magnetic field configurations that are obtained from simulation studies which can be useful for comparison with satellite data. Electric field models are in much poorer condition. One of the high priority objectives of this work is to obtain much-improved electric field models (see below). Simulation studies also provide electric field models, usually through an appropriate interpretation of the mean drift velocity that the simulation provides. We propose to work toward making it possible to incorporate recent magnetic and electric field models into the tool, as well as hooks for MHD descriptions of the electric and magnetic fields

2.2.2 Boundary Identifiers

One of the most important uses of UBK for particle detectors is the identification of boundaries. The last closed drift shell at some specified energy is often an easily identified feature in particle data sets. However, a local identification of such a feature with, say, the last closed drift shell, has not in the past been able to provide much information on the global aspect of such a boundary. The (U, B, K) analysis shows that such features can usually be identified with trajectories in the (U, B) plane that are tangent to the W = 0 curves. We propose to develop algorithms to be used with selected electric and magnetic field models so as to simplify the comparison of the features as seen in the data with what the models predict from the (U, B, K) treatment, much in the way that the SPOF displays magnetic mapping boundaries in its plots.

2.2.3 Using (U, B, K) to Identify Particle Trajectories Connecting Different Magnetospheric Spacecraft

The correlation of spectral features between widely separated spacecraft has remained a thorny problem for space physicists, mostly because of the dependence of the trajectories on energy and pitch-angle. The UBK formalism will allow the development of tools that permit the calculation of orbit intersections, so that ion trajectories can be found which connect the two spacecraft. This will enable particle detectors to become tracer experiments of the magnetosphere.

One of the major goals of ISTP is to use the observations from different spacecraft to gain understanding of magnetospheric processes. We believe that the (U, B, K) system provides a tool for comparing particle data from different spacecraft in a quantitative way. The key point is that for a given pair of spacecraft, depending on their location, there is in general a common set of particles which can be seen by each spacecraft. That is, there are particle trajectories which connect the spacecraft. These trajectories can be identified quite simply by means of the (U, B, K) system, provided of course that the electric and field models are reasonably accurate (see also section 2.1.3). If indeed these trajectories, characterized by their local energy and pitch angle at each spacecraft, can be identified, then the number of particles on each trajectory (i.e. their distribution functions) provides information on loss processes, scattering, etc, that have happened to the particles on their travel between the spacecraft. Conversely, features in the particle data seen by both spacecraft, such as injection fronts, minima at particular energies, etc., provide information on the accuracy of the field models.

We believe that this capability can be of great value for the analysis and comparison of data from different spacecraft. We propose to develop this idea in collaboration with ISTP investigators and to prepare and make available software tools for the identification of particle trajectories that can be seen by different pairs of spacecraft, based on their location and using particular models for the magnetospheric electric and magnetic fields. We note that the equations of drift motion (Section 1) also provide the means for obtaining the "times of flight" for the particles to travel between the different spacecraft. This would be useful for obtaining timing information on the propagation of events.

3 Deepening the Foundations and Broadening the Applications

The (U, B, K) approach was developed initially as an aid to analyzing data but we believe that it can be a powerful theoretical tool as well. However, the approximations that are involved in its formulation and the resulting limitations on its use need to be clearly understood. For example, the total energy as given in (1) for the particle at its mirror point neglects the kinetic energy due to the particle's drift motion. The drift energy is usually but not always small. The formulation also neglects scattering processes. The derivation of the technique assumes a steady state for both magnetic and electric fields, thus neglecting changing magnetic fields and induced electric fields which are important during active times. It neglects parallel electric fields, and the formulation is non-relativistic which limits its application to electrons with energies below $\sim$100 keV. We propose to examine how the technique could be extended so that it would apply in regions and situations where these limiting factors are at present significant. The goal here is to have a solid theoretical basis for the (U, B, K) approach, a wide range of regimes for its validity, and a clear understanding of any remaining limitations.

3.0.4 Drift Kinetic Energy

The kinetic energy of gyro- and bounce-motion is included in the kinetic energy term (because we deal with particles at their mirror point) but not the kinetic energy of the drift motion itself. Although this is a second order effect, it becomes important whenever the kinetic energy of the first order terms approaches zero. Thus the drift kinetic energy could be important for "cold" plasma when it is in regions where the drift velocity is large, as in boundary layers for example. This is important if we are to use the UBK transformation to discuss time-dependent processes such as charge-exchange or Coulomb drag losses. We propose to examine how the drift energy could be taken into account in regions where it may be significant.

3.1 Extension to Relativistic Regime

For the ring current energies described in the examples above, a non-relativistic Hamiltonian was sufficient to describe the ion energies. However, electrons begin showing relativistic effects around 100keV (which are important for the generation of AKR), so that significant errors begin to creep into a non-relativistic treatment even at ring current energies, not to mention the relativistic electrons responsible for satellite upsets. In addition, radiation belt models assume circular drift orbits, which are increasingly inaccurate below L<2 due to magnetic multipole moments. In order to calculate the diffusion on non-circular drift orbits with this method, a relativistic Hamiltonian approach is required to solve the radiation belt model. (Although outside the scope of the proposed work, we note that generalization of this technique to Jupiter's magnetosphere will facilitate understanding of the relativistic particles involved in both Jovian decametric and synchrotron radiation.)

We note that the papers mentioned above by *Northrop and Teller* (1960) and *Northrop* (1963) derive the relativistic forms for the particle invariants. It appears possible to use the relativistic form for the particle total energy to obtain a relation between the electrostatic potential and the magnetic field intensity for the particle mirror point in the (relativistic) K = constant surface. The trajectory of the mirror point follows a curved path instead of a straight line, but this may still be a very useful tool for the analysis of data. We propose to develop the (U, B, K) technique for relativistic particles.

3.2 A Hamiltonian description

An intriguing possibility is that the (U, B, K) coordinates, together with the particle magnetic moment, can be formulated within a Hamiltonian description, so that a canonical system of coordinates could be obtained. The advantage of this is that one then has a phase space to which Liouville s theorem can be applied and in which a phase space density, i.e. a distribution function, for the particle mirror points can be defined. A Hamiltonian formulation of the equations for a charged particle moving in an electromagnetic field can of course be constructed even in the relativistic limit, e.g. (*Goldstein*, 1953, *Jackson*, 1962), but the translation of this into a form that makes use of the adiabatic invariants is not at all straightforward (*Kruskal*, 1962). The authors of this proposal have been involved in this kind of study in the past (*Northrop and Teller*, 1960, *Northrop*, 1963, *Whipple et al.*, 1986) and believe that they are in a good position to examine how the (U, B, K) system may be related to a Hamiltonian formulation. We note that the magnetic moment is a "momentum" variable, and the variable K combines features of both position and momentum. All this needs to be clarified. We believe that it is important to base the (U, B, K) approach on a firm theoretical basis and we propose to do that.

A related problem here is to relate any "mirror point phase space density" to the ordinary particle phase space density and to obtain the appropriate expressions for finding moments such as particle densities, currents, pressures, etc., in terms of the former.

3.3 Extension to Include Time-Dependence

It would be extremely useful if the (U, B, K) approach could be extended to apply to time-dependent behavior. At present the formulation applies only if the scale for temporal changes is large compared to the drift times of particles, since the linear trajectories for particles in the (U, B) plane are valid only if both the electrostatic potential and the magnetic field remain constant during the particle drift. In addition, inductive electric fields cannot be treated since they are not described by an electrostatic potential. Since both the magnetic field and inductive electric field can be obtained from the vector potential, it may be possible to reformulate the (U, B, K) system in terms of the vector potential somehow. Inductive electric fields may at times be fairly well confined to limited spatial regions. If this is so, it may be possible to use the (U, B, K) system over most of the magnetosphere and to treat the limited time-dependent region in a different, more detailed way. We propose to study how to extend the (U, B, K) approach to time-dependent fields in order to be able to treat active periods when storms and sub-storms occur.

3.4 Extension to Include Parallel Electric Fields

As Alfvén predicted and later satellite measurements observed, parallel electric fields can exist along a field line despite the great mobility of cold electrons. In particular, the details of the pitch angle distributions (PAD) for electrons and ions can set up double layers along a field line (*Whipple*, 1977). These anisotropic PADs may arise naturally from the isotropic plasma sheet as it convects through the magnetosphere because of constraints on particle access imposed by the W = 0 surfaces. This thesis can be tested easily by UBK analysis. Thus there may be conditions under which parallel electric fields are generated continuously by magnetospheric convection. We believe that analysis of "zipper" distributions (*Fennell et al.*, 1981), which show a field-aligned component at E<10keV and a trapped, nose event at >20keV has its origin in this interplay of PADs and convection. However, the introduction of a parallel electric field changes a particle's mirror point, and therefore a modified treatment is required.

There are two possible approaches to incorporating parallel electric fields into the (U, B, K) approach: the first applies primarily to fields that are relatively confined spatially. In this case it should be possible to treat the field as a scattering center that changes the parallel velocity and perhaps also the magnetic moment. In the case where the parallel field is extended over some length along the magnetic field line, then the definition of the longitudinal invariant can be extended to include the parallel field as long as the magnetic moment remains constant. There is a problem with this in that it is difficult to see how to retain the modified invariant K (a function of position) which can only apparently be extracted from the true invariant J (which is a function of both position and velocity) when the parallel field vanishes. We propose to look at these possibilities to see how parallel electric fields might be incorporated into the (U, B, K) formalism in some reasonable manner. If this should be possible, then the highly dynamic auroral zone could be treated in addition to the inner magnetosphere.

Management

The work that is proposed here involves several investigators at three institutions. Dr. Whipple at the University of Washington is the Principal Investigator and will be responsible for directing the work. As a Co-investigator on the Polar Hydra instrument, Dr. Whipple will bring Hydra data to the project and will participate in its analysis. He will work closely with the other investigators on theoretical issues. He will also take the lead in contacting investigators on other ISTP instruments and seeking their collaboration to provide and analyze data which would be appropriate for carrying out this work.

Dr. Whipple's appointment at UW is as an Affiliate Professor. The University requires a Full Professor to be associated with each project, and so Prof. George Parks is a Co-investigator at no cost to this proposal. Prof. Parks will participate especially in his capacity as the Principal Investigator for the UltraViolet Imager (UVI) on the Polar spacecraft to provide and analyze imager data for comparison with particle data. He is also Co-Investigator on the Wind 3D Plasma instrument and will take the lead in collaborations with Wind investigators.

Dr. Sheldon is the lead investigator at Boston University and will take part both in the proposed theoretical studies and in data analysis, especially data from the CEPPAD instrument on Polar. Professor Harlan Spence, a CEPPAD Co Investigator at Boston University, is also a Co-Investigator on this proposal. Prof. Spence will also take part in the project at no cost to the investigation. His participation is partly due to the requirement for a Full Professor to be associated with the project, but his extensive experience in data analysis will be of great value.

Dr. Ted Northrop from Goddard Space Flight Center will participate primarily in theoretical studies. He will take the lead in examining the Hamiltonian formulation of the (U, B, K) approach and will work with the other investigators to study how to extend the formalism to the regimes where relativity, time-dependence, or parallel electric fields are important.

All three institutions have extensive computer facilities including workstations dedicated to GGS tasks. These computing facilities will be made available for the work proposed here at no additional cost. These facilities will be adequate for the work that is proposed.

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